In about 300 B.C. Eratosthenes, a librarian in Alexandria, Egypt, discovered how to measure the circumference of the Earth. This is one of the most astonishing achievements of ancient science. Only about 50 years after Aristotle described the evidence that supported the idea that the Earth is shaped like a sphere, Eratosthenes figured out how to measure its circumference. In this activity, your students will discover Eratosthenes' reasoning. Based on the evidence that Eratosthenes had available, they will calculate the Earth's circumference.

We suggest that you take into account the level of math understanding of your students. While making the calculation is easy, understanding the reasoning requires geometry skills. High school geometry students should have little difficulty. If you plan to introduce the activity with younger students, it is advisable first to introduce the two major mathematical concepts on which the activity depends: (1) You can measure the angle of a shadow formed by a stick as a fraction of a circle; and (2) parallel lines cut by a straight line create equal angles.

How Big Is the Earth?

Materials/Preparation

- Copy activity sheets for each student, using masters on pages 2 and 3: "How Big Is the Earth?—Part 1" and "How Big Is the Earth?—Part 2"
- Optional: 1 calculator per student

In Class—Eratosthenes' Method of Measuring the Earth

1. Ask your students to recall the previous activity, in which they learned one reason why people believed over 2,000 years ago that the Earth is shaped like a ball. Ask if anyone can suggest how to measure the size of the ball-shaped Earth. (Accept all answers.) Explain that in ancient times it was not possible to travel all the way around the Earth or into space. Nonetheless, a very intelligent librarian was able to figure out how large the Earth was.
2. Divide the class into teams of two or three students. Hand out the two activity sheets one at a time, allowing time for the students to read and discuss them in teams. Then lead a class discussion, answering questions as necessary. When discussing the distance between Alexandria and Syene, you may want to note that there is some dispute about the length of the unit of measurement (stadia) that Eratosthenes used in 300 B.C. According to J.L.E. Dryer, *A History of Astronomy from Thales to Kepler*, the most likely value of the Earth's circumference calculated by Eratosthenes was 24,662 miles. The modern value for the Earth's circumference is about 24,900 miles.
3. In conclusion, ask your class to imagine taking a 24,900 mile trip in a straight line on the surface of the Earth. They would travel in a great big circle all the way around the Earth and return to the same place! Eratosthenes knew this almost 2,300 years ago. Christopher Columbus did not have to prove that the Earth was shaped like a ball.

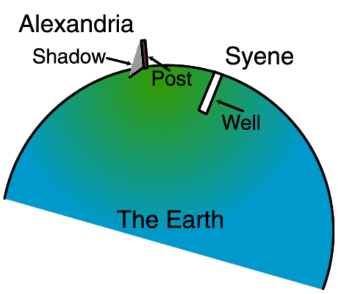
Name ____ Date ____________________

How Big Is the Earth? — Part 1

"My name is Eratosthenes. I'm a Greek scientist and librarian in the great library in Alexandria, Egypt. I have figured out a way to measure the distance around the ball-shaped Earth. Let me show you how you can do it too."

"I have read that at noon on the longest day of the year, the Sun's light shines directly down a well in Syene, a city that is several hundred miles to the south. (Locate Syene on the cross-section of the Earth on this page.) When I look at a vertical post in Alexandria at noon on the longest day of the year, the Sun's rays cast a shadow 1/8 the length of the post."

1. In the picture, draw the Sun, and show where it must be in order to shine directly down the well at noon in Syene and create a shadow in Alexandria.



2. If you see a vertical post in Syene at the same moment as the Sun shines directly down the well, does the post cast a shadow?
______yes ______no

3. Why do we see a shadow cast by the post at noon in Alexandria at the same time we see no shadow in Syene? What does this tell me about the shape of the Earth?

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Name ______________________________ Date ______________

How Big Is the Earth?—Part 2

Eratosthenes continues his story.

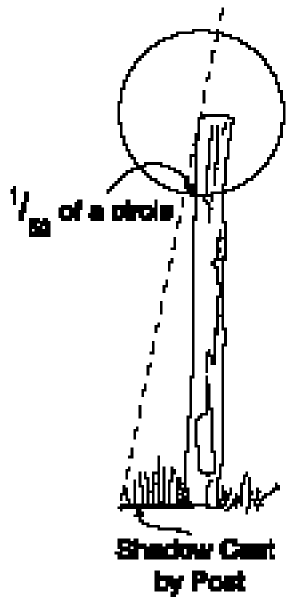
"One of my favorite books is about geometry, written by Euclid. It helped me find out how big the Earth is. At noon on the longest day of the year in Alexandria, the length of a shadow cast by a post was about 1/8 the length of the post. From Euclid's geometry, I found that the angle at the top of the post must have been about $^1/_{50}$ of a circle."

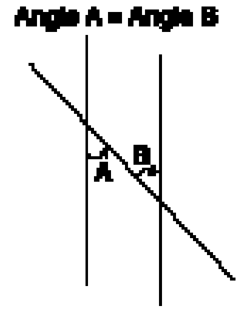
"From Euclid I also learned that if I draw two parallel lines with one straight line crossing both of them, certain angles are equal. In the drawing at left, angle A = angle B."

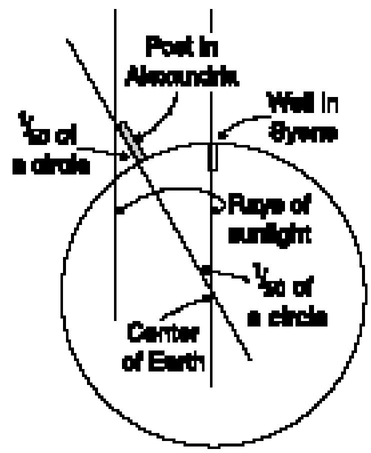
"One day I read a book which said that at noon on the longest day of the year, the sun shines straight down a well in Syene, several hundred miles south of Alexandria. Since the Sun is very far away, a ray of sunlight that reaches Alexandria is parallel to a ray of sunlight that reaches Syene."

"When I drew a diagram of the Earth I realized that the angle of the shadow cast by the vertical post in Alexandria equals an imaginary angle formed by the center of the Earth, Alexandria and Syene. Since that angle was $^1/_{50}$ of a circle, the distance between Alexandria and Syene must be $^1/_{50}$ of the distance around the Earth."

"Then I paid someone to measure the distance from Syene to Alexandria by walking from one city to the other, and counting his steps. He measured the distance to be 5,000 stadia [about 493 miles in modern terms]."







How Big Is the Earth? (give answers on the back of the paper)

1. If 493 miles (793 km) is $^1/_{50}$ of the way around the world, how many miles is it all the way around the world? (Show your calculations.)

____

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2. What is the circumference of the world by modern measurements? (Look it up!) How close was Eratosthenes' calculated measurement to the modern measurement?

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Key for How Big Is the Earth—Part 1

1. The Sun is directly overhead—straight over the city of Syene and the well.
2. When the Sun is directly overhead (see #1), a vertical post will not cast a shadow because the Sun is also directly over the post.
3. Eratosthenes sees a shadow in Alexandria because the Sun is not directly overhead in Alexandria. Alexandria is 493 miles (793 km), or about 7 degrees of latitude north of Syene, and the Sun can be directly overhead at only one place on the Earth at a time. Further, on a particular date, the Sun is directly overhead at noon for only one latitude around the globe, so places north and south of each other can never have the same shadow lengths at the same moment in time.

The Earth is shaped like a ball, and Eratosthenes knew it because there was a shadow from a vertical post at noon in Alexandria on the same day when there were no shadows at noon in Syene, a city south of Alexandria. On a flat Earth, all of the shadows would be the same at the same time of the day (in this case at noon).

Key for How Big Is the Earth—Part 2

How big is the Earth?

1. If 493 miles (or 793 km) is 1/50 of the way around the world, how many miles is it all the way around the world?

   50 x 1/50 = 1 whole circumference of the Earth. So the problem is solved by taking 50 times the distance of 493 miles between Alexandria and Syene.

   50 x 493 miles = 24,650 miles, or

   50 x 793 km = 39,650 km

This is Eratosthenes' estimate of the circumference of the Earth.

2. What is the circumference of the world by modern measurements?

   24,906 miles, or 40,074 km.

How close was Eratosthenes' calculated measurement to the modern measurement?

Modern circumference:	**24,906 miles**	**40,074 km**
Eratosthenes' circumference:	**24,650 miles**	**39,650 km**
The difference:	**256 miles**	**424 km**

For older students, you may wish to calculate the percentage of error, which may be found by dividing "the difference" by "the modern circumference":

256 miles ÷ 24,906 miles = .01 = 1% or
424 km ÷ 40,074 km = .01 = 1%